



2012 HSC Mathematics Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	B
2	D
3	C
4	A
5	C
6	D
7	C
8	A
9	B
10	B

Section II

Question 11 (a)

Criteria	Marks
• Correct answer	2
• Incorrect answer of the form $(2x \pm 1)(x \pm 3)$ or $(2x \pm 3)(x \pm 1)$	1

Question 11 (b)

Criteria	Marks
• Correct solution	2
• Obtains $x < 1$ or $x > -\frac{1}{3}$	1

Question 11 (c)

Criteria	Marks
• Correct solution	2
• Correct slope, or equivalent merit	1

Question 11 (d)

Criteria	Marks
• Correct answer	2
• Obtains $5(3 + e^{2x})^4$ or a product involving $(3 + e^{2x})$ and e^{2x} , or equivalent merit	1

Question 11 (e)

Criteria	Marks
• Correct answer	2
• Finds focal length or equivalent	1

Question 11 (f)

Criteria	Marks
• Correct solution	2
• Makes some progress	1

Question 11 (g)

Criteria	Marks
• Correct solution	3
• Correctly evaluates $\left(a \tan \frac{x}{2}\right)_0^{\frac{\pi}{2}}$, or equivalent merit	2
• Primitive of the form $a \tan \frac{x}{2}$, or equivalent merit	1

Question 12 (a) (i)

Criteria	Marks
• Correct answer	2
• Attempts to use the product rule	1

Question 12 (a) (ii)

Criteria	Marks
• Correct answer	2
• Attempts to use the quotient rule, or equivalent merit	1

Question 12 (b)

Criteria	Marks
• Correct primitive	2
• Obtains primitive involving $\log_e(x^2 + 6)$	1

Question 12 (c) (i)

Criteria	Marks
• Correct answer	2
• Makes some progress	1

Question 12 (c) (ii)

Criteria	Marks
• Correct answer	1

Question 12 (c) (iii)

Criteria	Marks
• Correct solution	2
• Makes substantial progress	1

Question 12 (d) (i)

Criteria	Marks
• Correct solution	3
• Makes significant progress in the use of Simpson's rule	2
• Attempts to use Simpson's rule	1

Question 12 (d) (ii)

Criteria	Marks
• Correct answer	1

Question 13 (a) (i)

Criteria	Marks
• Correct solution	2
• Finds A and B , or equivalent merit	1

Question 13 (a) (ii)

Criteria	Marks
• Correct solution	2
• Attempts to use the cosine rule, or equivalent merit	1

Question 13 (a) (iii)

Criteria	Marks
• Correct solution	3
• Makes substantial progress towards finding N	2
• Attempts to find an equation for the line CN , or equivalent progress	1

Question 13 (b) (i)

Criteria	Marks
• Correct solution	1

Question 13 (b) (ii)

Criteria	Marks
• Correct solution	3
• Correct primitive, or equivalent merit	2
• Writes a definite integral of the difference of the two functions, or equivalent merit	1

Question 13 (c) (i)

Criteria	Marks
• Correct answer	1

Question 13 (c) (ii)

Criteria	Marks
• Correct answer	1

Question 13 (c) (iii)

Criteria	Marks
• Correct solution	2
• Finds the probability that both marbles are white, or equivalent merit	1

Question 14 (a) (i)

Criteria	Marks
• Correct solution	3
• Correctly solves $f'(x) = 0$ and attempts to find the nature of the stationary points, or equivalent merit	2
• Attempts to solve $f'(x) = 0$, or equivalent merit	1

Question 14 (a) (ii)

Criteria	Marks
• Correct graph	2
• Correct general shape showing their stationary points, or equivalent merit	1

Question 14 (a) (iii)

Criteria	Marks
• Correct solution	1

Question 14 (a) (iv)

Criteria	Marks
• Correct solution	1

Question 14 (b)

Criteria	Marks
• Correct solution	3
• Correct primitive, or equivalent merit	2
• Significant progress to definite integral for the volume, or equivalent merit	1

Question 14 (c) (i)

Criteria	Marks
• Correct explanation	1

Question 14 (c) (ii)

Criteria	Marks
• Correct solution	1

Question 14 (c) (iii)

Criteria	Marks
• Correct solution	1

Question 14 (c) (iv)

Criteria	Marks
• Correct solution	2
• Attempts to solve a relevant exponential equation	1

Question 15 (a) (i)

Criteria	Marks
• Correct solution	2
• Attempts to sum a relevant geometric series	1

Question 15 (a) (ii)

Criteria	Marks
• Correct solution	1

Question 15 (b) (i)

Criteria	Marks
• Correct answer	1

Question 15 (b) (ii)

Criteria	Marks
• Correct answer	1

Question 15 (b) (iii)

Criteria	Marks
• Correct solution	2
• Correct primitive, or equivalent merit	1

Question 15 (b) (iv)

Criteria	Marks
• Correct solution	2
• Attempt to solve $\dot{x} = 0$, or equivalent merit	1

Question 15 (c) (i)

Criteria	Marks
• Correct expression	1

Question 15 (c) (ii)

Criteria	Marks
• Correct solution	2
• A correct expression for A_{300} involving M , or equivalent merit	1

Question 15 (c) (iii)

Criteria	Marks
• Correct solution	3
• Makes substantial progress in solving $A_n = 180\,000$	2
• Attempts to solve the equation $A_n = 180\,000$, or equivalent merit	1

Question 16 (a) (i)

Criteria	Marks
• Correct solution	2
• States one relevant geometric fact with reason, or equivalent merit	1

Question 16 (a) (ii)

Criteria	Marks
• Correct solution	2
• Obtains $\frac{b-x}{x} = \frac{x}{a-x}$, or equivalent merit	1

Question 16 (b) (i)

Criteria	Marks
• Correct solution	2
• Finds the slope of PT or the coordinates of T , or equivalent merit	1

Question 16 (b) (ii)

Criteria	Marks
• Correct solution	1

Question 16 (b) (iii)

Criteria	Marks
• Correct solution	2
• Finds OP , or equivalent merit	1

Question 16 (b) (iv)

Criteria	Marks
• Correct solution	3
• Correctly solves $\frac{dA}{d\theta} = 0$, or equivalent merit	2
• Correctly finds $\frac{dA}{d\theta}$, or equivalent merit	1

Question 16 (c) (i)

Criteria	Marks
• Correct solution	2
• Eliminates x or y from $y = x^2$ and $x^2 + (y - c)^2 = r^2$, or equivalent merit	1

Question 16 (c) (ii)

Criteria	Marks
• Correct solution	1



2012 HSC Mathematics 'Sample Answers'

When examination committees develop questions for the examination, they may write 'sample answers' or, in the case of some questions, 'answers could include'. The committees do this to ensure that the questions will effectively assess students' knowledge and skills.

This material is also provided to the Supervisor of Marking, to give some guidance about the nature and scope of the responses the committee expected students would produce. How sample answers are used at marking centres varies. Sample answers may be used extensively and even modified at the marking centre OR they may be considered only briefly at the beginning of marking. In a few cases, the sample answers may not be used at all at marking.

The Board publishes this information to assist in understanding how the marking guidelines were implemented.

The 'sample answers' or similar advice contained in this document are not intended to be exemplary or even complete answers or responses. As they are part of the examination committee's 'working document', they may contain typographical errors, omissions, or only some of the possible correct answers.

Section II**Question 11 (a)**

$$2x^2 - 7x + 3 = (2x - 1)(x - 3)$$

Question 11 (b)

$$|3x - 1| < 2$$

$$-2 < 3x - 1 < 2$$

$$-1 < 3x < 3$$

$$\text{Hence } -\frac{1}{3} < x < 1$$

Question 11 (c)

$$y = x^2, \quad \frac{dy}{dx} = 2x$$

slope of tangent at $x = 3$ is $2 \times 3 = 6$

$$\therefore 6 = \frac{y - 3^2}{x - 3} = \frac{y - 9}{x - 3}$$

$$\begin{aligned} \text{Hence the equation of the tangent is } y &= 6(x - 3) + 9 \\ &= 6x - 9 \end{aligned}$$

Question 11 (d)

$$\begin{aligned} y' &= 5(3 + e^{2x})^4 \times 2e^{2x} \\ &= 10e^{2x}(3 + e^{2x})^4 \end{aligned}$$

Question 11 (e)

$$x^2 = 16(y - 2) = 4 \cdot 4(y - 2), \text{ so } a = 4$$

the vertex is at $(0, 2)$, so the focus is at $(0, 2 + 4) = (0, 6)$

Question 11 (f)

Area of the sector is given by $A = \frac{\theta}{2}r^2$

$$\text{ie } 50 = \frac{\theta}{2}r^2$$

$$= \frac{\theta}{2} \cdot 6^2$$

$$= \frac{\theta}{2} \cdot 36$$

$$= 18\theta$$

$$\therefore \theta = \frac{50}{18}$$

$$\text{now, } l = r\theta$$

$$= 6 \times \frac{50}{18}$$

$$\text{length of arc} = \frac{50}{3} \text{ cm}$$

Question 11 (g)

$$\int_0^{\frac{\pi}{2}} \sec^2 \frac{x}{2} dx = \left[2 \tan \frac{x}{2} \right]_0^{\frac{\pi}{2}}$$

$$= 2 \tan \frac{\pi}{4} - 2 \tan 0$$

$$= 2 \times 1 - 0$$

$$= 2$$

Question 12 (a) (i)

$$\begin{aligned} y' &= \log_e x + (x-1) \frac{1}{x} \\ &= \log_e x + 1 - \frac{1}{x} \end{aligned}$$

Question 12 (a) (ii)

$$\begin{aligned} y' &= \frac{-x^2 \sin x - 2x \cos x}{x^4} \\ &= \frac{-(x \sin x + 2 \cos x)}{x^3} \end{aligned}$$

Question 12 (b)

$$\begin{aligned} \int \frac{4x}{x^2 + 6} dx &= 2 \int \frac{2x}{x^2 + 6} dx \\ &= 2 \log_e (x^2 + 6) + C \end{aligned}$$

Question 12 (c) (i)

Every row has two tiles more than the previous row and the first row has three tiles.

It is an arithmetic sequence and $T_{20} = 3 + 19 \times 2$

$$= 41$$

ie There are 41 tiles in row 20.

Question 12 (c) (ii)

The number of tiles for the 20 rows is

$$\begin{aligned} S_{20} &= \frac{20}{2}(3 + T_{20}) \\ &= 10(3 + 41) \\ &= 440 \end{aligned}$$

Question 12 (c) (iii)

We want $\frac{n}{2}(3 + T_n) = 200$,

where $T_n = 3 + 2(n - 1) = 2n + 1$.

Hence

$$\frac{n}{2}(3 + 2n + 1) = 200$$

$$\text{ie } n(n + 2) = 200$$

$$n^2 + 2n - 200 = 0$$

$$\therefore n = \frac{-2 \pm \sqrt{4 + 800}}{2}$$

$$= -1 \pm \sqrt{201}$$

$$= 13.1774$$

Hence Jay can make 13 complete rows.

Question 12 (d) (i)

If $f(x)$ denotes depth at distance x from the river bank then, by Simpson's rule, the approximate area is:

$$A = \frac{3}{8}(f(0) + 4f(3) + 2f(6) + 4f(9) + f(12))$$

$$= 1(0.5 + 4 \times 2.3 + 2 \times 2.9 + 4 \times 3.8 + 2.1)$$

$$\therefore \text{area} = 32.8 \text{ m}^2$$

Question 12 (d) (ii)

Volume through the cross-section in 10 seconds is

$$(32.8 \times 0.4 \times 10) \text{ m}^3 = 131.2 \text{ m}^3$$

Question 13 (a) (i)

Coordinates of A : $y = 0$, $2x = 8$, so
 $x = 4$

ie $A(4, 0)$

coordinates of B : $x = 0$, $y = 8$

ie $B(0, 8)$

$$\begin{aligned} \text{By Pythagoras' theorem } AB &= \sqrt{4^2 + 8^2} \\ &= \sqrt{80} \\ &= 4\sqrt{5} \end{aligned}$$

Question 13 (a) (ii)

By the cosine rule

$$AC^2 = AB^2 + BC^2 - 2AB \times BC \cos(\angle ABC)$$

$$25 = 80 + 65 - 2 \times 4\sqrt{5} \times \sqrt{65} \cos(\angle ABC)$$

$$40\sqrt{13} \cos(\angle ABC) = 120$$

$$\cos(\angle ABC) = \frac{3}{\sqrt{13}}$$

Hence $\angle ABC \approx 33.69^\circ$,

so the angle is 34° to the nearest degree.

Question 13 (a) (iii)

Slope of AB is -2 , so the slope of CN is $\frac{1}{2}$.

Equation of CN is $\frac{1}{2} = \frac{y-4}{x-7}$,

$$\text{so } y = \frac{1}{2}(x-7) + 4$$

$$= \frac{x}{2} + \frac{1}{2}.$$

The coordinates of N are obtained by the intersection of AB and CN :

$$y = -2x + 8 = \frac{1}{2}x + \frac{1}{2}$$

$$-4x + 16 = x + 1$$

$$15 = 5x$$

$$3 = x$$

Since $y = -2x + 8$

$$y = -6 + 8$$

$$= 2$$

Hence N has coordinates $(3, 2)$

Question 13 (b) (i)

For the x -coordinate at the intersection of the two parabolas

$$x^2 - 3x = 5x - x^2$$

$$2x^2 - 8x = 0$$

$$2x(x-4) = 0$$

$$x = 0, 4$$

so $x = 4$ is the x -coordinate of point A .

Question 13 (b) (ii)

Area is given by

$$\int_0^4 (5x - x^2) - (x^2 - 3x) dx$$

$$= \int_0^4 8x - 2x^2 dx$$

$$= \left[4x^2 - \frac{2}{3}x^3 \right]_0^4$$

$$= 64 - \frac{128}{3}$$

$$= \frac{192 - 128}{3}$$

$$\text{area} = \frac{64}{3} \text{ units}^2$$

Question 13 (c) (i)

$$\frac{3}{5} \times \frac{3}{7} = \frac{9}{35}$$

Question 13 (c) (ii)

$$\text{Complement of (i)} : 1 - \frac{9}{35} = \frac{26}{35}$$

Question 13 (c) (iii)

Probability of 2 red + probability of 2 white:

$$\frac{9}{35} + \frac{2}{5} \times \frac{4}{7} = \frac{17}{35}$$

Question 14 (a) (i)

$$\begin{aligned} f'(x) &= 12x^3 + 12x^2 - 24x \\ &= 12x(x^2 + x - 2) \\ &= 12x(x + 2)(x - 1) \end{aligned}$$

Hence the stationary points are at $x = 0$, $x = 1$, $x = -2$

$$\begin{aligned} \text{Now } f(0) &= 0, \quad f(1) = 3 + 4 - 12 \quad \text{and} \quad f(-2) = 316 - 48 - 124 \\ &= -5 \qquad \qquad \qquad = -32 \end{aligned}$$

Hence the stationary points are

$$(-2, -32), (0, 0), (1, -5)$$

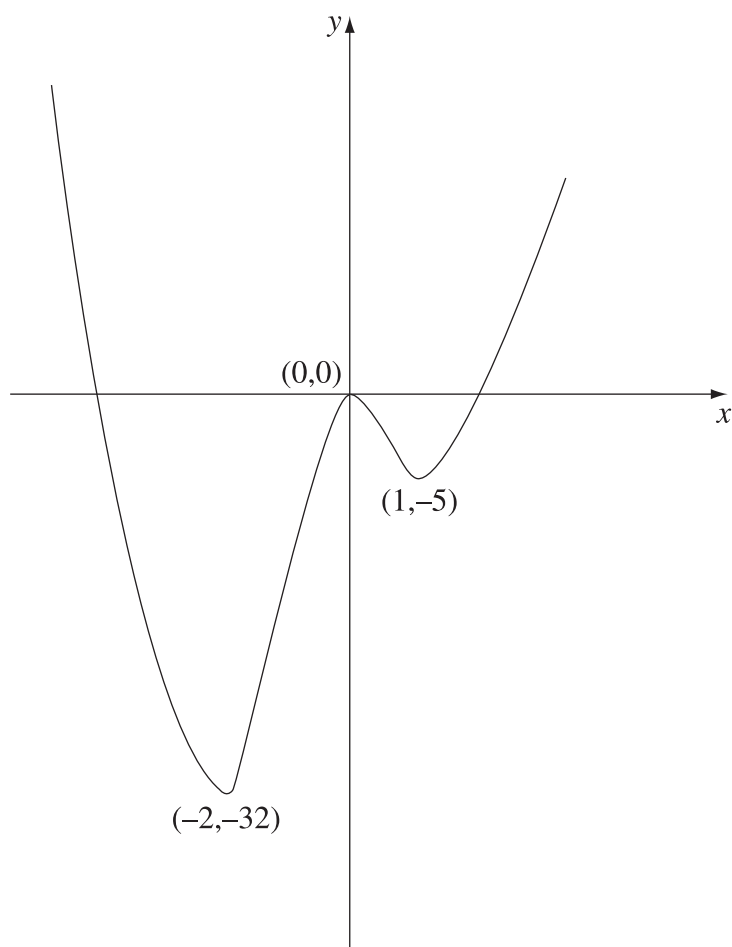
$$\text{Now } f''(x) = 12(3x^2 + 2x - 2)$$

$$f''(0) = -24 < 0, \text{ so } (0, 0) \text{ is a maximum}$$

$$f''(1) = 12 \times 3 > 0, \text{ so } (1, -5) \text{ is a minimum}$$

$$f''(-2) = 12 \times 6 > 0, \text{ so } (-2, -32) \text{ is a minimum}$$

Question 14 (a) (ii)



Question 14 (a) (iii)

$f(x)$ is increasing for $-2 < x < 0$ or for $x > 1$

Question 14 (a) (iv)

k is the vertical shift of the graph of f .

To make sure the equation has no solution (ie the new graph should not cut the x -axis) move the graph up by the smallest minimum, so $k > 32$.

Question 14 (b)

Volume is given by $V = \int \pi y^2 dx$

$$\begin{aligned} V &= \pi \int_0^1 \frac{9}{(x+2)^4} dx \\ &= 9\pi \int_0^1 (x+2)^{-4} dx \\ &= \left[-\frac{9\pi}{3} (x+2)^{-3} \right]_0^1 \\ &= -3\pi (3^{-3} - 2^{-3}) \\ &= -3\pi \left(\frac{1}{27} - \frac{1}{8} \right) \end{aligned}$$

$$\therefore \text{Volume} = \frac{19\pi}{72} \text{ units}^3$$

Question 14 (c) (i)

$$\begin{aligned} N(20) &= 1000e^{20k} = 2000 \\ e^{20k} &= 2 \\ 20k &= \ln 2 \\ k &= \frac{\ln 2}{20} \approx 0.0347 \end{aligned}$$

Question 14 (c) (ii)

$$\begin{aligned} N(120) &= 1000e^{120k} \\ &= 1000 e^{120 \times 0.0347} \end{aligned}$$

Number of bacteria $\approx 64\,328$

Question 14 (c) (iii)

$$\frac{dN}{dt} = kN, \text{ so from (ii)}$$

$$\frac{dN}{dt} = 0.0347 \times 64\,328 \approx 2232 \text{ when } t = 120$$

rate of change ≈ 2232 bacteria/minute

Question 14 (c) (iv)

At $t = 0$ $N = 1000$

Find t so that $100\,000 = 1000e^{kt}$

$$100 = e^{kt}$$

$$\ln 100 = kt$$

Hence
$$t = \frac{\ln 100}{k}$$

$$= \frac{\ln 100}{0.0347}$$

time ≈ 132.7 minutes

Question 15 (a) (i)

Length in cm is

$$\begin{aligned}
 &10 + 10 \times 0.96 + 10 \times 0.96^2 + \dots + 10 \times 0.96^9 \\
 &= 10(1 + 0.96 + \dots + 0.96^9) \\
 &= 10\left(\frac{1 - 0.96^{10}}{1 - 0.96}\right) \\
 &\approx 83.79
 \end{aligned}$$

Question 15 (a) (ii)

Since $0.96 < 1$ the limiting sum $10(1 + 0.96 + 0.96^2 + \dots)$ exists.

$$\text{The limiting sum is } 10\left(\frac{1}{1 - 0.96}\right) = \frac{10}{0.04} = 250$$

As 250 cm < 300 cm, a strip of length 3 m is sufficient.

Question 15 (b) (i)

$$\begin{aligned}
 \text{Initial velocity is } \dot{x}(0) &= 1 - 2\cos 0 \\
 &= -1 \text{ m/s}
 \end{aligned}$$

Question 15 (b) (ii)

$$\ddot{x} = 2\sin t = 0 \text{ if } t = 0, \pi, 2\pi, \dots$$

The first maximum velocity is at $t = \pi$

$$\begin{aligned}
 \dot{x}(\pi) &= 1 - 2\cos \pi \\
 &= 3 \text{ m/s}
 \end{aligned}$$

Question 15 (b) (iii)

$$\begin{aligned}x &= \int \dot{x} \, dt = \int 1 - 2\cos t \, dt \\&= t - 2\sin t + C\end{aligned}$$

We are given that $x(0) = 3$, so

$$x(0) = -2\sin 0 + C = 3$$

Hence $C = 3$ and the displacement is

$$x = t - 2\sin t + 3$$

Question 15 (b) (iv)

The particle is at rest if $\dot{x} = 0$, so

$$\dot{x} = 1 - 2\cos t = 0$$

$$\text{ie } \cos t = \frac{1}{2}$$

$$\text{Hence } t = \frac{\pi}{3}$$

$$\begin{aligned}\text{The displacement is } x\left(\frac{\pi}{3}\right) &= \left(\frac{\pi}{3} - 2\sin\frac{\pi}{3} + 3\right) \text{ metres} \\&= \left(\frac{\pi}{3} - \sqrt{3} + 3\right) \text{ metres}\end{aligned}$$

Question 15 (c) (i)

$$\begin{aligned}A_2 &= [360\,000(1 + 0.005) - M](1 + 0.005) - M \\&= 360\,000(1.005)^2 - M(1 + 1.005)\end{aligned}$$

Question 15 (c) (ii)

Generalising from (i)

$$\begin{aligned} A_n &= 360\,000(1.005)^n - M(1 + 1.005 + \dots + 1.005^{n-1}) \\ &= 360\,000(1.005)^n - M \frac{(1.005^n - 1)}{(1.005 - 1)} \end{aligned}$$

We require $A_{300} = 0$, so

$$\begin{aligned} 360\,000(1.005)^{300} &= M \frac{(1.005^{300} - 1)}{(1.005 - 1)} \\ M &= \frac{360\,000(1.005)^{300} \cdot 0.005}{(1.005^{300} - 1)} \approx 2319.50 \end{aligned}$$

Question 15 (c) (iii)

We want to find the smallest n so that $A_n < 180\,000$

$$\begin{aligned} 360\,000(1.005)^n - M \frac{(1.005^n - 1)}{0.005} &= 180\,000 \\ 360\,000(1.005)^n - 463\,900(1.005^n - 1) &= 180\,000 \\ 103\,900(1.005)^n &= 283\,900 \\ (1.005)^n &= 2.7324 \end{aligned}$$

$$\text{Hence } n = \frac{\log 2.7324}{\log 1.005} = 201.5$$

After 202 months $\$A_n$ will be less than $\$180\,000$ for the first time.

Question 16 (a) (i)

$EF \parallel CD$ since $CDEF$ is a rhombus

$ED \parallel FC$ since $CDEF$ is a rhombus

$\angle FEB = \angle DAE$ (corresponding angles, $EF \parallel CA$)

$\angle FBE = \angle DEA$ (corresponding angles, $ED \parallel BC$)

Hence $\triangle EBF$ is similar to $\triangle AED$ since two (and therefore all) angles are equal.

Question 16 (a) (ii)

Using that $\triangle EBF$ is similar to $\triangle AED$,

$$\frac{x}{a-x} = \frac{b-x}{x} \quad (\text{corresponding sides of similar triangles})$$

$$x^2 = (b-x)(a-x)$$

$$x^2 = ba - ax - bx + x^2$$

$$0 = ba - x(a+b)$$

$$x = \frac{ab}{a+b}$$

Question 16 (b) (i)

T has coordinates $(\cos\theta, \sin\theta)$

The line OT has slope $\frac{\sin\theta}{\cos\theta}$

Hence the line PT perpendicular to OT has slope $-\frac{\cos\theta}{\sin\theta}$ and passes through T .

Hence the equation of PT is:

$$-\frac{\cos\theta}{\sin\theta} = \frac{y - \sin\theta}{x - \cos\theta}$$

$$-x\cos\theta + \cos^2\theta = y\sin\theta - \sin^2\theta$$

$$x\cos\theta + y\sin\theta = \cos^2\theta + \sin^2\theta$$

$$= 1$$

Question 16 (b) (ii)

Q is the point of intersection of the line $y = 1$ with the line from (i).

Hence the x -coordinates of Q satisfies

$$x \cos \theta + 1 \sin \theta = 1$$

$$x = \frac{1 - \sin \theta}{\cos \theta}$$

The length of BQ is $\frac{1 - \sin \theta}{\cos \theta}$

Question 16 (b) (iii)

Area of trapezium is given by

$$A = \frac{1}{2} OB (OP + BQ)$$

P is on the line $x \cos \theta + y \sin \theta = 1$ with $y = 0$,

$$\text{so } x = \frac{1}{\cos \theta} \quad \text{ie } OP = \frac{1}{\cos \theta}$$

$$OB = 1 \text{ and from (ii) } BQ = \frac{1 - \sin \theta}{\cos \theta}$$

$$\begin{aligned} \therefore A &= \frac{1}{2} \left(\frac{1}{\cos \theta} + \frac{1 - \sin \theta}{\cos \theta} \right) \\ &= \frac{1}{2} \left(\frac{2 - \sin \theta}{\cos \theta} \right) \\ &= \frac{2 - \sin \theta}{2 \cos \theta} \end{aligned}$$

Question 16 (b) (iv)

Differentiate area with respect to θ :

$$\begin{aligned}\frac{dA}{d\theta} &= \frac{d}{d\theta} \left(\frac{2 - \sin\theta}{2\cos\theta} \right) \\ &= \frac{-\cos^2\theta + (2 - \sin\theta)\sin\theta}{2\cos^2\theta} \\ &= \frac{2\sin\theta - (\cos^2\theta + \sin^2\theta)}{2\cos^2\theta} \\ &= \frac{2\sin\theta - 1}{2\cos^2\theta}\end{aligned}$$

Need to solve $2\sin\theta - 1 = 0$

$$\text{ie } \sin\theta = \frac{1}{2}$$

Hence $\theta = \frac{\pi}{6}$ is a critical point

If $\theta \rightarrow \frac{\pi}{2}$ then the area of the trapezium becomes very large: $A \rightarrow \infty$

If $\theta = 0$, then $\frac{dA}{d\theta} = -\frac{1}{2} < 0$, so the area is decreasing.

As there is only one stationary point it must be minimum.

Hence $\theta = \frac{\pi}{6}$ gives the minimum area.

Question 16 (c) (i)

Find the points of intersection of the parabola $y = x^2$ and a circle $x^2 + (y - c)^2 = r^2$:

$$y + (y - c)^2 = r^2$$

$$y + y^2 - 2cy + c^2 = r^2$$

$$y^2 + (1 - 2c)y + c^2 - r^2 = 0$$

The circle is tangent if there is precisely one solution, so the discriminant has to vanish.

$$(1 - 2c)^2 - 4(c^2 - r^2) = 0$$

$$(1 - 2c)^2 = 4(c^2 - r^2)$$

$$1 - 4c + 4c^2 = 4c^2 - 4r^2$$

$$4c = 1 + 4r^2 \text{ as required}$$

Question 16 (c) (ii)

y must be positive to be a solution since the circle is inside the parabola.

As the discriminant is zero

$$y = -\frac{1}{2}(1 - 2c) \geq 0$$

$$\text{so } 1 - 2c \leq 0$$

$$\frac{1}{2} \leq c$$

If $c = \frac{1}{2}$ there is only one point, so $c > \frac{1}{2}$.

Mathematics

2012 HSC Examination Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	1.1	P3
2	1	1.1	P3
3	1	9.2	P4
4	1	10.1, 10.4	H7
5	1	6.5	P4
6	1	5.2, 5.3, 13.1	P4, H5
7	1	12.1, 12.2	H3
8	1	4.4	P4
9	1	12.5	H3, H5
10	1	11.1, 11.2	H5

Section II

Question	Marks	Content	Syllabus outcomes
11 (a)	2	1.3	P3
11 (b)	2	1.4	P3
11 (c)	2	10.7	P6, P7
11 (d)	2	8.9, 12.5	P7, H3
11 (e)	2	9.5	H5, H9
11 (f)	2	13.1	H5
11 (g)	3	11.2, 13.6	H5
12 (a) (i)	2	8.8, 12.5	P7, H3
12 (a) (ii)	2	8.8, 13.5	P7, H5
12 (b)	2	12.5	H5
12 (c) (i)	2	7.5	H5
12 (c) (ii)	1	7.5	H5
12 (c) (iii)	2	7.5	H5
12 (d) (i)	3	11.3	H8
12 (d) (ii)	1	14.1	H4
13 (a) (i)	2	6.5	P4
13 (a) (ii)	2	5.5	P4
13 (a) (iii)	3	6.2, 6.3	H5
13 (b) (i)	1	1.4	P4

Question	Marks	Content	Syllabus outcomes
13 (b) (ii)	3	11.4	H8
13 (c) (i)	1	3.2, 3.3	H5
13 (c) (ii)	1	3.2, 3.3	H5
13 (c) (iii)	2	3.2, 3.3	H5
14 (a) (i)	3	10.2	H6
14 (a) (ii)	2	10.5	H6
14 (a) (iii)	1	10.1	H6
14 (a) (iv)	1	10.5, 10.6	H2
14 (b)	3	11.2, 11.4	H8
14 (c) (i)	1	12.2, 14.2	H3, H4
14 (c) (ii)	1	14.2	H3, H4
14 (c) (iii)	1	14.2	H3, H4
14 (c) (iv)	2	14.2	H3, H4
15 (a) (i)	2	7.2, 7.5	H5
15 (a) (ii)	1	7.3, 7.5	H5
15 (b) (i)	1	14.3	H4, H5
15 (b) (ii)	1	14.3	H4, H5
15 (b) (iii)	2	13.6, 14.3	H4, H5
15 (b) (iv)	2	14.3	H4, H5
15 (c) (i)	1	7.5	H5
15 (c) (ii)	2	7.5	H5
15 (c) (iii)	3	7.5, 12.2	H3, H5
16 (a) (i)	2	2.3	H2, H5
16 (a) (ii)	2	2.3	H2, H5
16 (b) (i)	2	5.1, 6.2	P4, H2, H5
16 (b) (ii)	1	5.1, 6.3	H2, H5
16 (b) (iii)	2	2.3, 6.3, 6.8	P4, H2, H5
16 (b) (iv)	3	10.6, 13.5	H2, H5
16 (c) (i)	2	6.8, 9.2, 9.3	P4, H2, H4, H5
16 (c) (ii)	1	6.8, 9.3	P4, H2, H4, H5